Lateral Inhibition Enhances the Detectability of a Pure Tone in the Presence of Background Noise

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1 Introduction

Lateral inhibition in the auditory system is said to enhance the detectability of pure tones in a noisy environment. What does this mean? All information about our acoustic environment is coded in the spike-trains of the auditory nerve. An ideal classifier can decide whether there is a pure tone in the noise with minimal error rate. So the *ideal* classifier does not need any preprocessing of the data. Lateral inhibition as a preprocessing step can only be useful if non ideal classifiers are used afterwards.

To decide whether lateral inhibition can enhance detectability we have to make some hypotheses about these non ideal classifiers. The simplest classifier that detects a pure tone in noise is the following. Determine the neuron that has fired most often. If there have been more spikes than a certain threshold, detect a pure tone at the best frequency of this neuron.

In a simulation we show that lateral inhibition can reduce the error probability of this detector.

2 The Model

The input layer has excitatory axons contacting the output layer. Neurons of the output layer inhibit each other laterally.

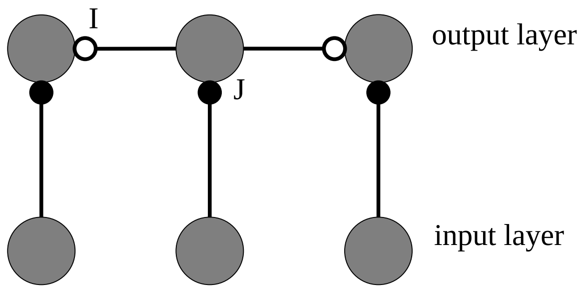


Figure 1: Model of the connections between the neurons. Neurons are indicated by gray circles, axons by lines, excitatory synapses by filled circles and inhibitory synapses by open circles.
Note that not all inhibitory connections are shown.

Neurons are modeled as spike response neurons [2, 3, 4].

The synaptic potential h_{syn} of neuron i in the output layer is

$$h_{syn}(i,t) = J \sum_{t_f^{in}(i)} \varepsilon(t - t_f^{in}(i)) - \sum_{j \neq i} I(i,j) \sum_{t_f(j)} \varepsilon(t - t_f(j)).$$

Here $t_f^{in}(i)$ are the firing times of neuron i in the input layer, $t_f(i)$ are the firing times of neuron i in the output layer and J is the strength of the feed-forward coupling.

The strength $I(i,j)$ of the lateral inhibition between the neurons i and j in the output layer was taken to be

$$I(i,j) = I e^{-(i-j)^2/d^2}.$$

The refractory potential h_{ref} of neuron i is

$$h_{ref}(i,t) = \sum_{t_f(i)} \eta(t - t_f(i)).$$

The membrane potential is

$$h(i,t) = h_{syn}(i,t) + h_{ref}(i,t).$$

The neuron fires whenever

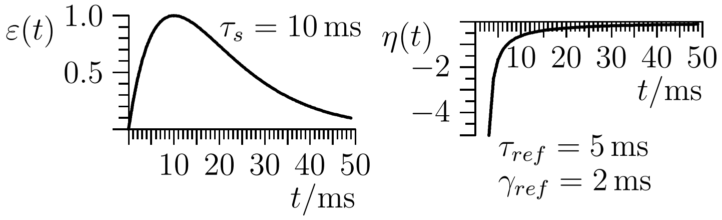
$$h(i,t) > \theta.$$

For $\varepsilon(t)$ and $\eta(t)$

$$\varepsilon(t) = \frac{t}{\tau_s} e^{1-t/\tau_s} \quad \text{for} \quad t \geqslant 0$$

$$\eta(t) = \begin{cases} -\infty & \text{for} \quad 0 < t \leqslant \gamma_{ref} \\ -\frac{\tau_{ref}}{t-\gamma_{ref}} & \text{for} \quad t > \gamma_{ref} \end{cases}$$

have been assumed.



d	θ	τ_s	τ_{ref}	γ_{ref}
3	1	10 ms	5 ms	2 ms

Table 1: Constants used in the simulation.

3 The Simulation

Noise was simulated by neurons in the input layer firing at a mean firing rate of 50 Hz. The pure tone was simulated by a single neuron in the input layer firing at 150 Hz. All neurons in the input layer fired according to a Poisson process.

The detector decides within 100 ms whether there is a pure tone in the stimulus or not. To do so it determines the neuron that has fired most often. If this is less than a certain threshold, the detector votes for noise only. Otherwise the detector votes for an additional pure tone. The threshold of the detector is chosen optimal, i.e. such that the error probability is minimal.

To measure the minimum error probability of the detector the following probability distributions are needed.

- In case the pure tone is present in addition to noise: The probability that the neuron with best frequency at the pure tone fires exactly n times within 100 ms.
- In case that only noise is present: The probability that the neuron which fires most often fires exactly n times within 100 ms.

The error in the detection rate of the threshold filter with optimal threshold was measured in dependence of the strength of the lateral inhibition I and the feed-forward coupling J.

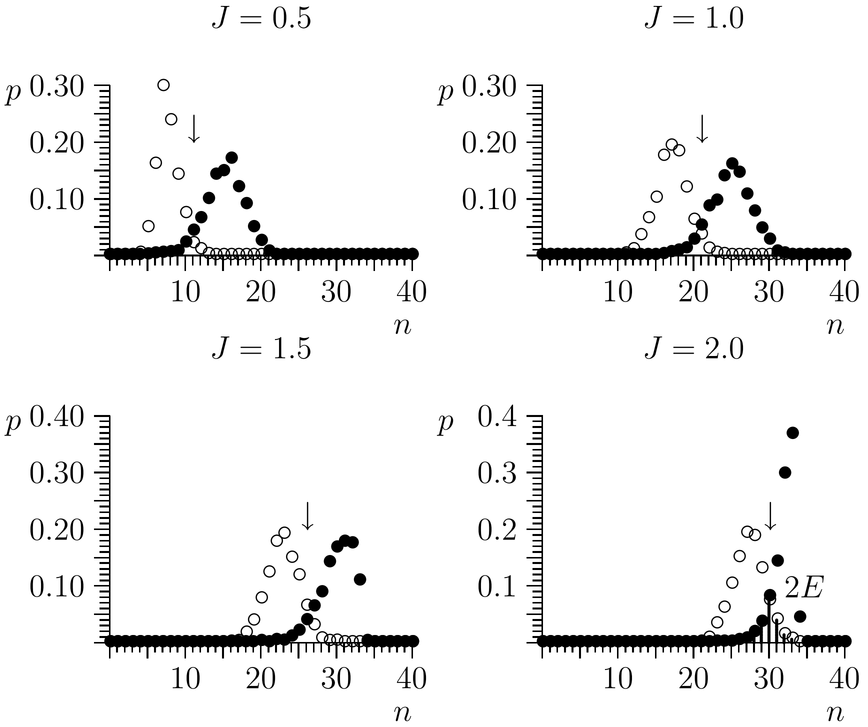


Figure 2:
○ Probability p that the neuron that has fired most often has fired exactly n times during 100 ms with only noise as stimulus.
• Probability p that the neuron with best frequency at the pure tone has fired exactly n times during 100 ms with noise and an additional pure tone as stimulus.
↓ marks the threshold of the optimal threshold detector.
The error probability E is the area under ○ left of the threshold plus the area under • right of the threshold divided by 2.
The strength J of the feed-forward coupling varies as indicated. Here the strength of lateral inhibition is $I = 0$. 100 s have been simulated.

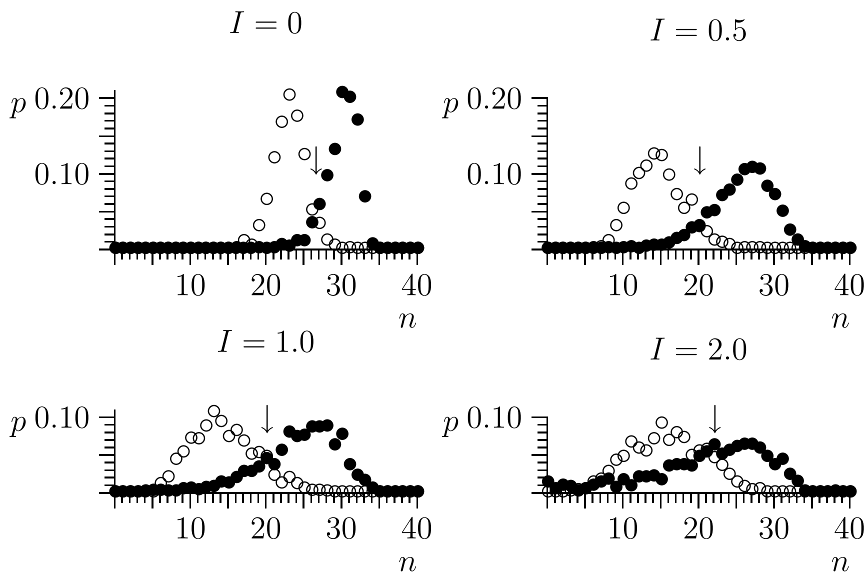


Figure 3: The strength I of the lateral inhibition varies as indicated. Here the strength of the feed-forward coupling is $J = 1.5$. 100 s have been simulated.

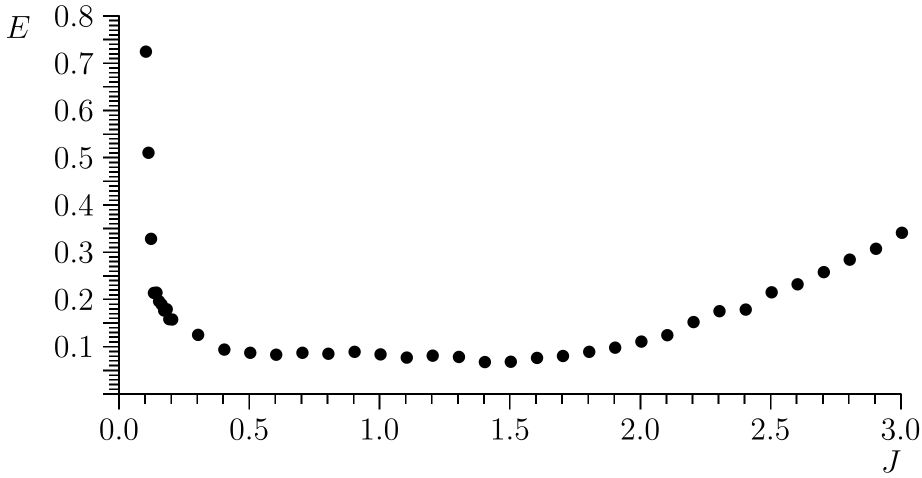


Figure 4: Error probability E in dependence of the strength J of the feed-forward coupling. Here the strength of the lateral inhibition is $I = 0$. 100 s have been simulated.

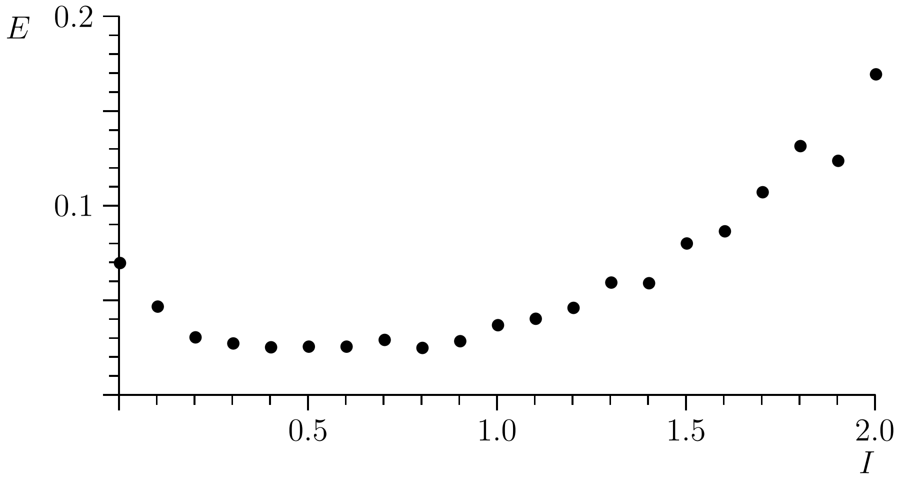


Figure 5: Error probability E in dependence of the strength I of the lateral inhibition. Here the strength of the feed-forward coupling is $J = 1.5$. 100 s have been simulated.

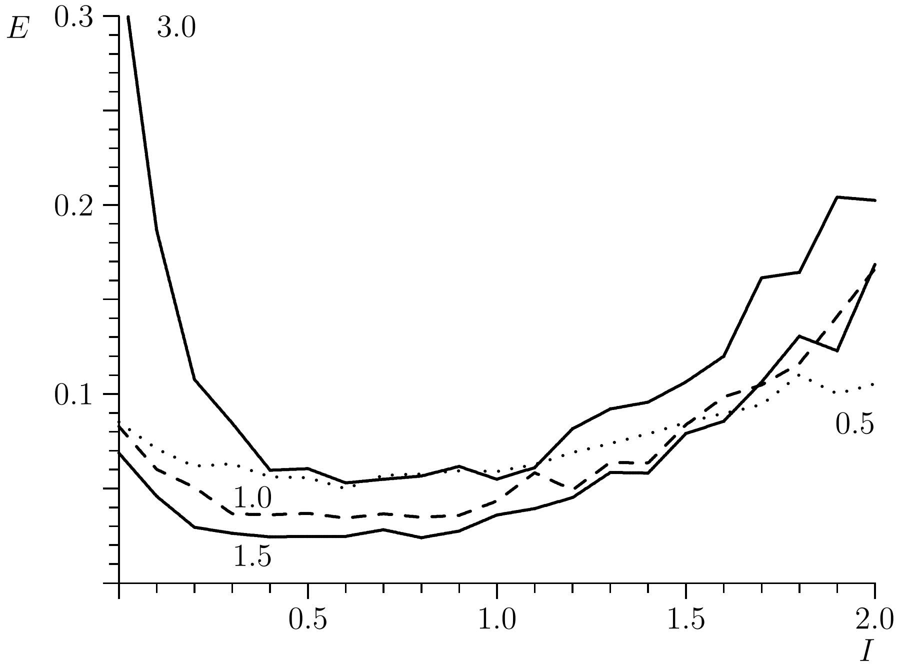


Figure 6: Error probability E in dependence of the strength I of the lateral inhibition. Different feed-forward couplings J are indicated. 100 s have been simulated.

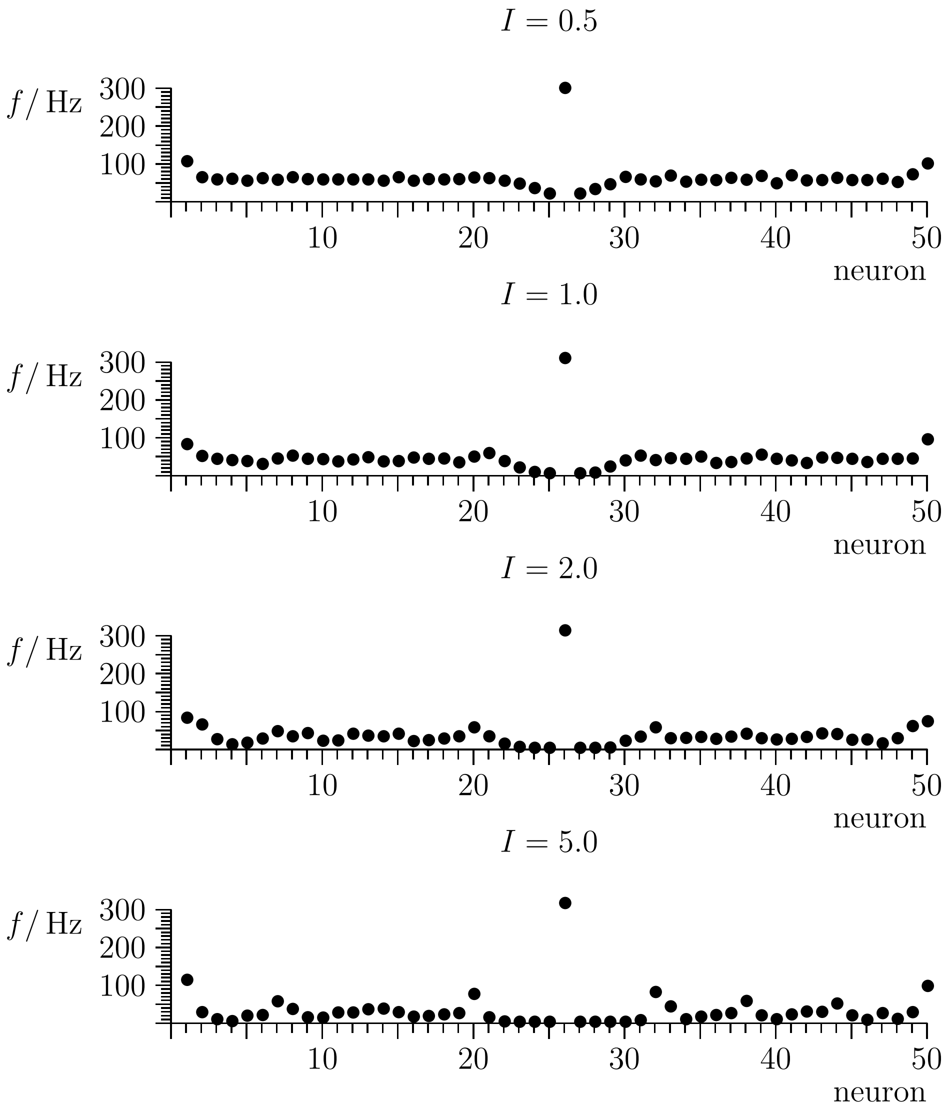


Figure 7: Firing rates of the neurons of the output layer. Stimulus was noise in addition to a pure tone at neuron 26. The different strengths of the lateral inhibition I are indicated.
Here the strength of the feed-forward coupling is $J = 1.5$. 100 s have been simulated.

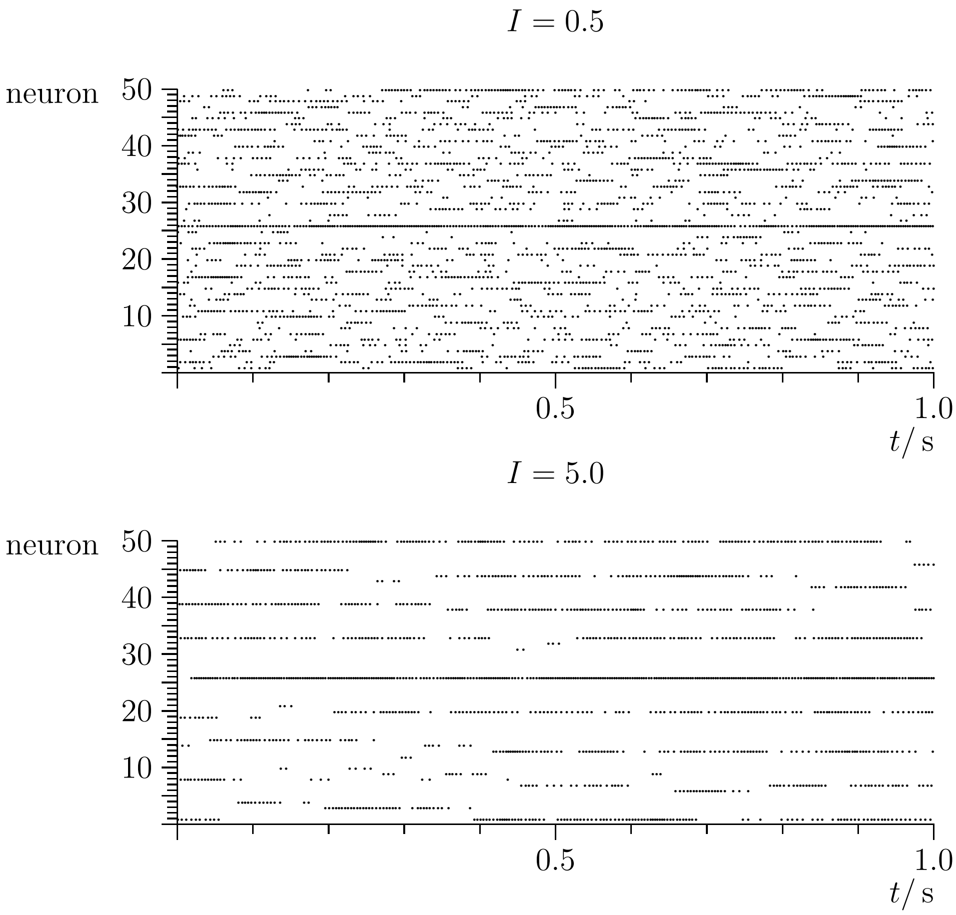


Figure 8: Stimulus is the same as in Figure 7. Every point indicates an action potential of a neuron at time t. The different strengths of the lateral inhibition I are indicated.
Here the strength of the feed-forward coupling is $J = 1.5$. 100 s have been simulated.
Note that the threshold detector has do decide which neuron has fired most often after an interval of only 100 ms.

4 Results

- Lateral inhibition can reduce the error probability of the threshold detector with optimal threshold.
- The exact strength of lateral inhibition is rather uncritical.
- Too much inhibition causes high firing rates in single, equally spaced neurons.

References

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