Mechanik der Kontinua

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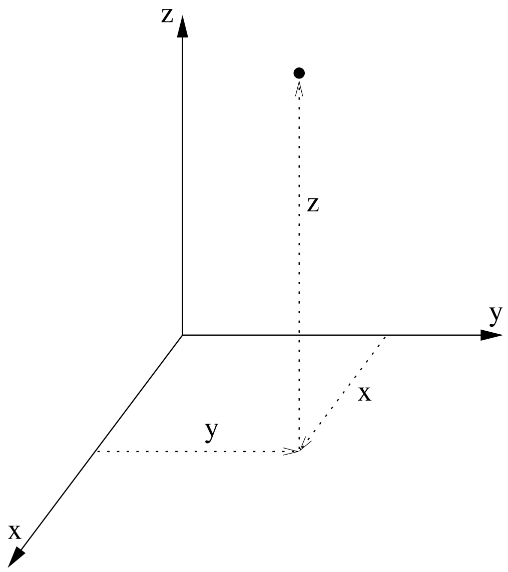
Bei den angegebenen Koordinaten handelt es sich um rechtwinklige, *normierte* Koordinaten.

1. **Kartesische Koordinaten** (x, y, z)



$$\mathrm{grad}\, f = \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix}$$

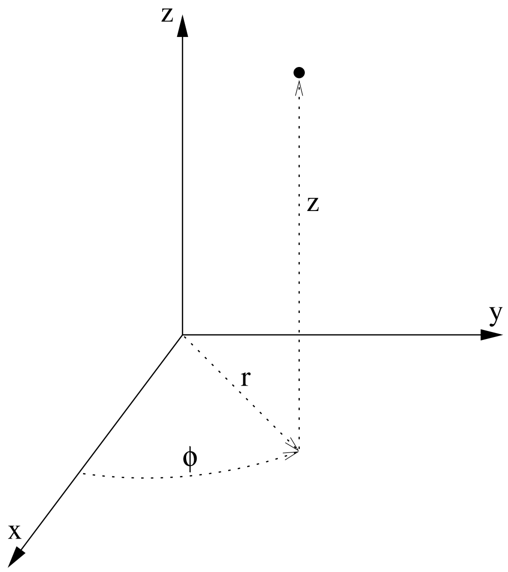
$$\mathrm{div}\, \vec{v} = \partial_x v_x + \partial_y v_y + \partial_z v_z$$

$$\mathrm{rot}\, \vec{v} = \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix}$$

$$\Delta f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\Delta \vec{v} = \begin{pmatrix} \partial_x^2 v_x + \partial_y^2 v_x + \partial_z^2 v_x \\ \partial_x^2 v_y + \partial_y^2 v_y + \partial_z^2 v_y \\ \partial_x^2 v_z + \partial_y^2 v_z + \partial_z^2 v_z \end{pmatrix}$$

2. **Zylinderkoordinaten** (r, ϕ, z)



$$\mathrm{grad}\, f = \begin{pmatrix} \partial_r f \\ \frac{1}{r}\partial_\phi f \\ \partial_z f \end{pmatrix}$$

$$\mathrm{div}\, \vec{v} = \frac{1}{r}\partial_r(r v_r) + \frac{1}{r}\partial_\phi v_\phi + \partial_z v_z$$

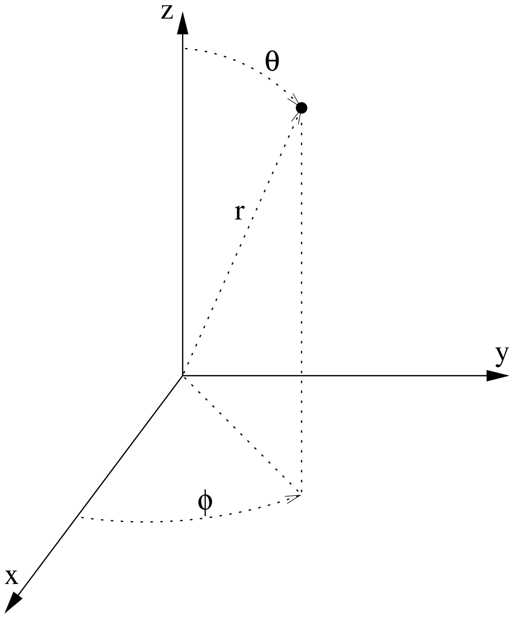
$$\mathrm{rot}\, \vec{v} = \begin{pmatrix} \frac{1}{r}\partial_\phi v_z - \partial_z v_\phi \\ \partial_z v_r - \partial_r v_z \\ \frac{1}{r}(\partial_r(r v_\phi) - \partial_\phi v_r) \end{pmatrix}$$

$$\Delta f = \frac{1}{r}\partial_r(r\partial_r f) + \frac{1}{r^2}\partial_\phi^2 f + \partial_z^2 f$$

$$\Delta \vec{v} = \begin{pmatrix} \partial_r^2 v_r + \frac{1}{r^2}\partial_\phi^2 v_r + \partial_z^2 v_r + \frac{1}{r}\partial_r v_r - \frac{2}{r^2}\partial_\phi v_\phi - \frac{v_r}{r^2} \\ \partial_r^2 v_\phi + \frac{1}{r^2}\partial_\phi^2 v_\phi + \partial_z^2 v_\phi + \frac{1}{r}\partial_r v_\phi + \frac{2}{r^2}\partial_\phi v_r - \frac{v_\phi}{r^2} \\ \partial_r^2 v_z + \frac{1}{r^2}\partial_\phi^2 v_z + \partial_z^2 v_z + \frac{1}{r}\partial_r v_z \end{pmatrix}$$

Bitte wenden.

3. **Kugelkoordinaten** (r, θ, ϕ)



$$\text{grad } f = \begin{pmatrix} \partial_r f \\ \frac{1}{r}\partial_\theta f \\ \frac{1}{r\sin\theta}\partial_\phi f \end{pmatrix}$$

$$\text{div } \vec{v} = \frac{1}{r^2}\partial_r(r^2 v_r) + \frac{1}{r\sin\theta}\partial_\theta(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\partial_\phi v_\phi$$

$$\text{rot } \vec{v} = \begin{pmatrix} \frac{1}{r\sin\theta}(\partial_\theta(v_\phi\sin\theta) - \partial_\phi v_\theta) \\ \frac{1}{r}\left(\frac{1}{\sin\theta}\partial_\phi v_r - \partial_r(r v_\phi)\right) \\ \frac{1}{r}(\partial_r(r v_\theta) - \partial_\theta v_r) \end{pmatrix}$$

$$\Delta f = \frac{1}{r^2}\partial_r(r^2\partial_r f) + \frac{1}{r^2\sin\theta}\partial_\theta(\sin\theta\partial_\theta f) + \frac{1}{r^2\sin^2\theta}\partial_\phi^2 f$$

$$\Delta\vec{v} = \begin{pmatrix} \frac{1}{r}\partial_r^2(r v_r) + \frac{1}{r^2}\partial_\phi^2 v_r + \frac{1}{r^2\sin^2\phi}\partial_\theta^2 v_r + \frac{\cot\phi}{r^2}\partial_\phi v_r - \frac{2}{r^2}\partial_\phi v_\phi - \frac{2}{r^2\sin\phi}\partial_\theta v_\theta - \frac{2v_r}{r^2} - \frac{2\cot\phi}{r^2}v_\phi \\ \frac{1}{r}\partial_r^2(r v_\phi) + \frac{1}{r^2}\partial_\phi^2 v_\phi + \frac{1}{r^2\sin^2\phi}\partial_\theta^2 v_\phi + \frac{\cot\phi}{r^2}\partial_\phi v_\phi - \frac{2}{r^2}\frac{\cot\phi}{\sin\phi}\partial_\theta v_\theta + \frac{2}{r^2}\partial_\phi v_r - \frac{v_\phi}{r^2\sin^2\phi} \\ \frac{1}{r}\partial_r^2(r v_\theta) + \frac{1}{r^2}\partial_\phi^2 v_\theta + \frac{1}{r^2\sin^2\phi}\partial_\theta^2 v_\theta + \frac{\cot\phi}{r^2}\partial_\phi v_\theta + \frac{2}{r^2\sin\phi}\partial_\theta v_r + \frac{2\cot\phi}{r^2\sin\phi}\partial_\theta v_\phi - \frac{v_\theta}{r^2\sin^2\phi} \end{pmatrix}$$

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